Indian Statistical Institute
Back Paper Examination
Differential Topology - MMath II

Max Marks: 100

Time: 180 minutes.

Give proper and complete justification(s) for your answers.

(1) Show that $S^n = \{(x_0, \ldots, x_n) : x_i \in \mathbb{R}, \sum_i x_i^2 = 1\}$ is a manifold. Describe the tangent space at $x \in S^n$. [10]

(2) Define the notion of a critical point of a smooth function. Show that on a compact manifold every smooth real valued function has a critical point. Find the critical points of the function $f : S^1 \longrightarrow \mathbb{R}$ defined by $f(x, y) = xy$. [2+8+4]

(3) Define the term : *embedding*. Show that the map $f : \mathbb{R} \longrightarrow \mathbb{R}^2$ defined by

$$f(t) = \left(\frac{e^t + e^{-t}}{2}, \frac{e^t - e^{-t}}{2}\right)$$

is an embedding. [2+8]

(4) Show that the set $SL_2(\mathbb{R})$ of 2×2 matrices A with determininat $+1$ is a manifold. Describe the tangent space to $A \in SL_2(\mathbb{R})$. [20]

(5) Let $f : \mathbb{R}^2 - 0 \longrightarrow R^2$ be defined by

$$f(x, y) = \left(\frac{x}{\sqrt{x^2 + y^2}}, \frac{y}{\sqrt{x^2 + y^2}}\right).$$

Is f transversal to S^1? Justify. [15]

(6) State and prove the determinant theorem. [10]

(7) Compute $H^i_{dR}(S^1)$, $i = 0, 1$. [15]

(8) Let ν be any 1-form on S^1 with nonzero integral. Show that if ω is any other 1-form, then there is a constant c such that $\omega - c\nu$ is exact. [6]